Exercise 93

A soccer stadium holds 62,000 spectators. With a ticket price of \$11, the average attendance has been 26,000. When the price dropped to \$9, the average attendance rose to 31,000. Assuming that attendance is linearly related to ticket price, what ticket price would maximize revenue?

Solution

Start by writing a formula for the average attendance. Two points on the line are $(11, 26\,000)$ and $(9, 31\,000)$. The slope of the line is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{31\,000 - 26\,000}{9 - 11} = \frac{5\,000}{-2} = -2500$$

Use the point-slope formula with either of the two points to get the equation of the line.

$$y - 26\,000 = -2500(x - 11)$$

$$y - 26\,000 = -2500x + 27\,500$$

$$y = -2500x + 53\,500$$

The revenue generated is the ticket price x times the number of people that attend y.

$$\begin{aligned} R &= xy \\ &= x(-2500x + 53\,500) \\ &= -2500x^2 + 53\,500x \end{aligned}$$

Complete the square to write the quadratic function in vertex form.

$$\begin{aligned} R &= -2500\left(x^2 - \frac{107}{5}x\right) \\ &= -2500\left[\left(x^2 - \frac{107}{5}x + \frac{107^2}{10^2}\right) - \frac{107^2}{10^2}\right] \\ &= -2500\left[\left(x - \frac{107}{10}\right)^2 - \frac{107^2}{10^2}\right] \\ &= -2500\left(x - \frac{107}{10}\right)^2 + 286\,225 \end{aligned}$$

Therefore, the maximum revenue is $R = \$286{,}225$, which occurs when $x = \frac{107}{10} = \$10.70$.